



清華大學  
Tsinghua University

Interdisciplinary Investigation

# “Complex (soft) hydrodynamics”

From **single paradigms** to **X-paradigm**

Yunfan Huang

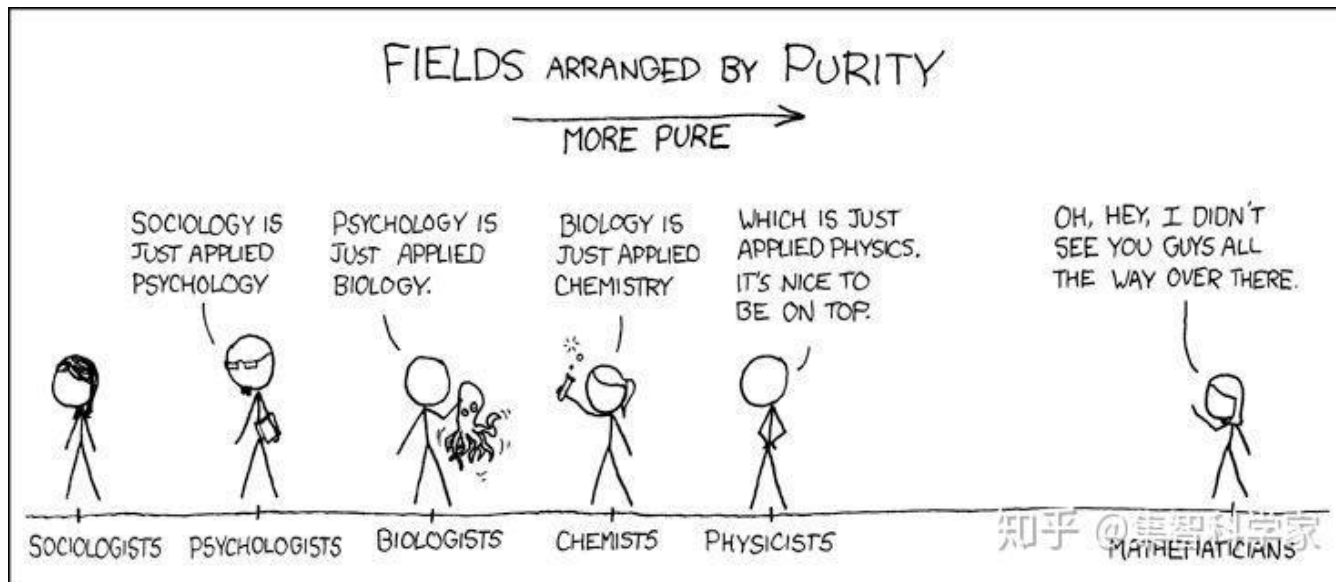
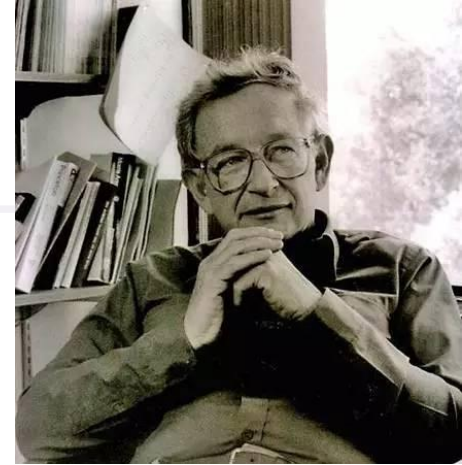
2024.12.18



# “MORE IS DIFFERENT”

- P.W. Anderson (1923-2020, USA; 1977 Nobel-Phys)

“for their fundamental theoretical investigations of the electronic structure of magnetic and disordered systems” (shared with Mott, van Vleck)



**Emergence** (涌现论/演生论) v.s. **Reductionism** (还原论)

1. P.W. Anderson. More is different: broken symmetry and the nature of the hierarchical structure of science, *Science*, 1972
2. 段远源, 专业基础课《高等工程热力学》, 文献翻译作业

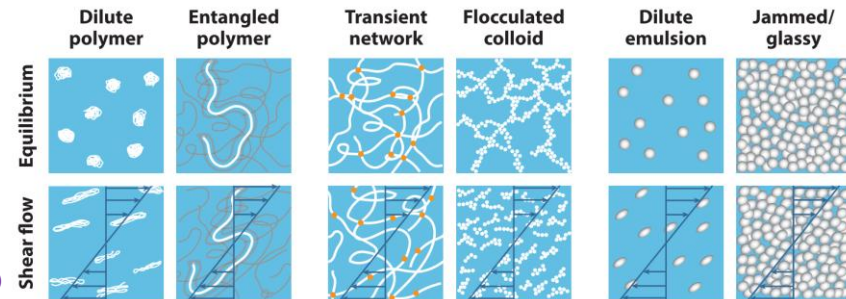
**Emergence ... of what? What can be actually observed and measured?**

找到长时间可观测的、可复现的规律[宏观守恒量, 模式/斑图]

# When hydrodynamics meets “complex”

➤ What is **Hydrodynamics**? Why we need it?

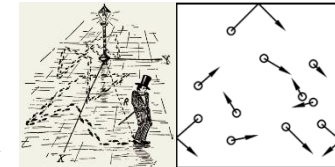
- ❑ quasi-particles as collective excitations
- ❑ long-wavelength (small wavevector) limit
- ❑ observables describing long-distance prop



- One way to develop a **macroscopic** theory with densities of physical quantities and the corresponding currents, is to combine
  - **continuity equations** (manifesting conservation laws), with
  - **thermodynamic arguments** (“constitutive relations”), between the macroscopic currents and the external bias to close the equations, to identify
    - how the entropy of the system responds to **local density fluctuations of the conserved quantities**
    - which requires the total entropy production rate to be non-negative
- They are **phenomenological** since they provide no means of calculating the coefficients in the constitutive relations, which is justified **at distances that are much larger than any “microscopic” scattering length scales**, the condition that is very often satisfied in experiments.

1. Narozhny, Hydrodynamics approach to 2D electron systems, 2022
2. 朗道, 场论、流体动力学、物理动理学; 谢多夫, 连续介质力学
3. Ewoldt, Designing complex fluids, *Annal Rev Fluid Mech*, 2022

random  
walk



short-range  
interaction

For “**strongly-correlated**” systems, does the hydrodynamic description still hold?

粒子之间存在长程关联[电磁作用、物质波相干性、界面交联/位阻作用…]



# Contents

- **From simple to complicated: find connections and differences**
  - ***“Where to start ... with something new”***
  - “Decouple” charge correlation: learn from TWO figures
  - Personal experience A: Electrolyte transport at interfaces
  - Personal experience B: Transport of correlated electron systems
- Complex hydrodynamics as X-paradigm: What, Why, How, and Which?
  - Example: Emergence of quantum hydrodynamics
  - New physics: “More SCALES at INTERFACE is Different”
  - Hallmark: Merging kinetic behaviors into hydrodynamics

朗道的工作有多重要？



拉格朗日的忧郁

知势榜科学与工程领域成长力榜答主

永公街的小魚 等 205 人赞同了该回答

要理解朗道的重要性，我们换到Anderson+视角

# “Where to start ... with something new”



- P.W. Anderson (1923-2020, USA; 1977 Nobel-Phys)
  - ❑ I think most people entering research find that by far the most difficult question is where to start, especially when confronted with something that is actually new. This, ... , is the kind of question a book like this should be designed to answer.
  - ❑ Many books are simply compendia of methods that have already been used or of techniques for calculating a little better something that is already understood. I am writing this immediately after the experience of having been confronted by the new phases of He; faced with such a genuinely novel problem it is far more important to have some idea of what the relevant questions are than it is to do any one calculation with great accuracy or rigor.
  - ❑ This is one of the reasons why I suggest that the two most important principles of condensed matter physics for our purposes are, first, **broken symmetry**, which tells us that what the order parameter is and what symmetry it breaks are the most vital questions; and, second, the **continuity principle**, which tells us to search for the right simple problem when confronted with a complicated one. To my way of thinking, detailed perturbation methods, and even Green's function and fluctuation-dissipation ideas, are somewhat less important, because they emphasize computation rather than understanding.

1. Anderson. *Basic notions of condensed matter physics*, 1984.
2. 朗道, 统计物理学 (I, II)、物理动力学.

为什么物理学本科专业不学流体力学?



已关注

有没有从统计物理出发, 推导流体力学基本理论的工作?

外行, 今天聊起来突然发问\_(x)\_ 各位大佬勿喷, 希望能学到新东西

已关注

编辑回答

邀请回答

好问题 13

4 条评论

分享

field  
dynamics  
**particle**

convection? interface?  
instability? pattern?  
viscosity? vortex?

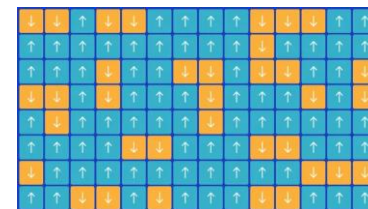
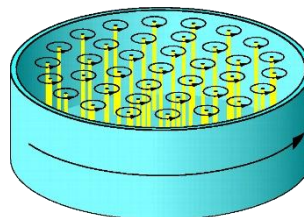
## How to choose the order parameter and adiabatic approximation?

找到模式(及转换)的核心控制量[物理量(无量纲数)] 从熟悉的系统过渡[极限情况做起]



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- Complex hydrodynamics as X-paradigm: What, Why, How, and Which?
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# Challenge I: Two typical correlated systems



## Electrolytes in solutions

- Nature
  - ✓ **dissolved**
    - hydrodynamic
    - chemical
- Correlation
  - ✓ steric
  - ✓ **electrostatic**
  - ✓ flow-induced

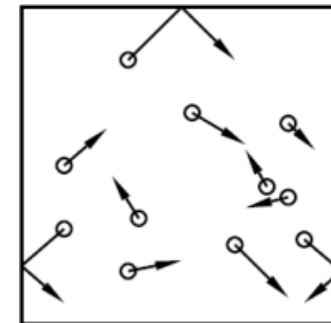
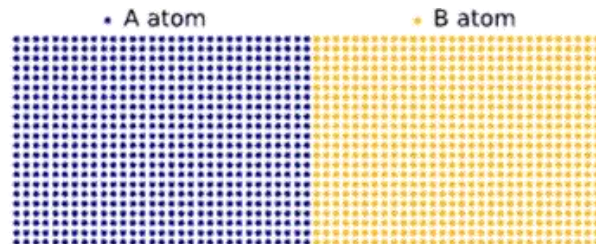
## Electrons in solids

- Nature
  - ✓ **fermion statistics**
  - ✓ **lattice regulation**
  - ✓ wave nature
- Correlation
  - ✓ coherent
  - ✓ **electro-**  $(e, I)^2$
  - ✓ magneto-  $(\uparrow, \downarrow)$

random  
walk



**Familiar ones:** rarefied gas transport (molecule)



short-range  
interaction

**WHY** diffusion decomposition of  
ion transport in solution still stands?

- ☐ Advection
- ☐ Diffusion
- ☐ **Electro-migration**

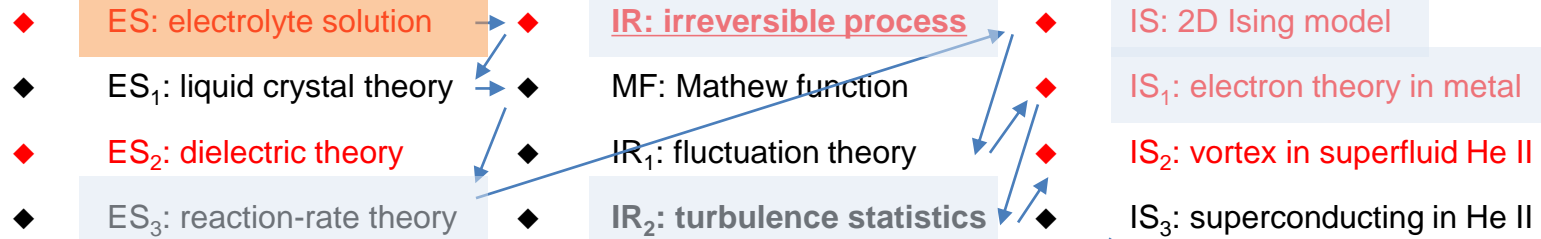
**WHY** particle kinetic description of  
electron transport still stands?

- ☐ **Excitation**
- ☐ Propagation
- ☐ **Scattering**

# Two figures: Onsager & Landau



## ➤ L. Onsager (1903-1976, USA; 1968 Nobel-Chem)



Extra topics: ice's electrical theory, radiochemistry, molecular biology (life in the early days)

## ➤ L.D. Landau (1908-1968, Russia; 1962 Nobel-Phys)

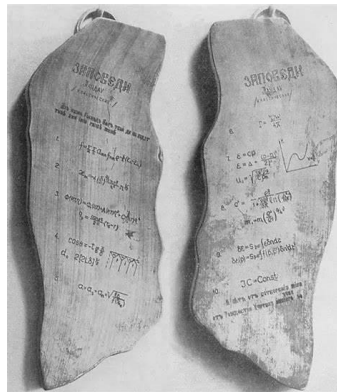
(QM) 量子力学的密度矩阵和统计物理学, 1927

(Mag) 自由电子抗磁性的理论, 1930

(Mag) 二级相变的研究, 1936-1937

(Mag) 铁磁性磁畴理论和反铁磁性解释, 1935

(CM) 超导体的混合态理论, 1934



The "Ten Commandments" (朗道十诫)

1937, 原子核的几率理论 (PP)

**1940-1941, 氦II超流性量子理论 (CM)**

1954, 基本粒子的电荷约束理论 (PP)

1956, 费米液体的量子理论 (CM)

1957, 弱相互作用的 CP 不变性 (PP)



理论物理学教程 (朗道十卷)

**Contributions:** Symmetry (ES-IR) & its breaking (IS; Mag-PP), correlation function (ES) & quasi-particle (CM-QM)

【找到模式(及转换)的核心控制量[物理量(无量纲数)]】   【从熟悉的系统过渡[极限情况做起]】   page8





# Onsager's theory on electrolyte in solution

**WHY** diffusion decomposition of ion transport in solution stands?

- ❑ Advection
- ❑ Diffusion
- ❑ Electro-migration

**Zur Theorie der Elektrolyte. I.**  
Von Lars Onsager.

§ 1. Einleitung. Debye und Hückel haben vor einiger Zeit den Einfluß der interionischen Kräfte auf die thermodynamischen Eigenschaften sowie auf die Leitfähigkeit von Elektrolyten theoretisch berechnet<sup>1)</sup>. Die Theorie der thermodynamischen Eigenschaften leistet nun gewissermaßen mehr als diejenige der Leitfähigkeit. Es wird nämlich z. B. für die Abhängigkeit der osmotischen Koeffizienten von der Konzentration der für hinreichend verdünnte Lösungen gültige Ausdruck:

$$g = 1 - \alpha \sqrt{c} \quad (1)$$

abgeleitet, wobei im Koeffizienten  $\alpha$  außer Temperatur, Wärmehinhalten des Ionen und Dielektrizitätskonstante der Lösung nur universelle Konstanten vorkommen. Der Koeffizient  $\alpha$  ist durch die folgenden Formeln ausgedrückt:

$$\Lambda = \Lambda_0(1 - \alpha \sqrt{c})$$

$$f_1 = 1 - k \sqrt{c} \quad (2)$$

1) Diese Zeitschrift 34, 185, 305, 1935, Fig. 1.

in einem gewissen Abstand vom Ion; wir bezeichnen denselben mit  $a$ , und nehmen entsprechend der zweiten Näherung von Debye und Hückel an, daß die Ladung der Ionen gleich  $4\pi e a$  ist.

und es wird nach Debye und Hückel die von ihnen eingeführte, der Wurzel aus der Verdünnung proportionale mittlere Dichte der Ionenatmosphäre gleich  $\frac{1}{2} \sqrt{c}$  gesetzt. Wir erhalten dann, bei der äußeren Feldstärke  $\mathcal{E}$ , für die Volumkraft:

**Total force balance, including electromigration & Stokes (viscous)**

des Ions grenzenden Flüssigkeitskeilchen sollen dieselbe Geschwindigkeit haben. Die totale Kraft auf das Ion soll gleich  $\mathcal{F}$  sein.

**Der Elektrolyte. I.** Physik. Zeitschr. XXVII, 1926.

Die Gleichungen von Stokes lauten nun, wenn  $v$  = Geschwindigkeit,  $p$  = Druck,  $\eta$  = Viskosität der Flüssigkeit:

$$\eta \operatorname{rot} \operatorname{rot} v = - \operatorname{grad} p + \vec{\sigma} \quad (4)$$

$$\operatorname{div} v = 0$$

Wir dürfen immer das Stromsystem in zwei Teile zerlegen:

$$v = v_1 + v_2$$

derart, daß  $v_1$  dem Fall

$$\mathcal{E}_1 = \mathcal{E}$$

$$\vec{\sigma}_1 = 0$$

$v_2$  dem Fall

$$\mathcal{E}_2 = 0$$

entsprechen. dann zulässig, nicht mit sich vereinigen lassen. Cesset gilt.

Was wir Geschwindigkeit deshalb nicht um System  $v_1$ , sondern System  $v_2$  dem Ion  $i$  der Kraft  $\mathcal{F}_i$  erteilt, und suchen weiter den  $\mathcal{F}_i$ , den das Kräftesystem  $\vec{\sigma}$  zur Geschwindigkeit des Ions liefert.

Dafür suchen das Stromsystem  $\mathcal{E}_2$ , das von den zwischen den Abständen  $r$  und  $r + dr$  vom Ion angreifenden Kräften herrührt. Wir gelangen sofort zum Ziel, wenn wir die Resultate von Stokes über die Bewegung der Kugel heranziehen. Nach Stokes bewegt sich bekanntlich eine Kugel vom Radius  $r$  unter Einwirkung der Kraft  $\vec{P}$  mit der Geschwindigkeit

$$\frac{P}{6\pi\eta r}$$

durch die Flüssigkeit. Die pro Flächeneinheit von der Flüssigkeit auf die Kugel übertragene Kraft, die sich aus Druck und Reibungskraft zusammensetzt, ist dabei gleichmäßig über die Kugelfläche verteilt, und für alle Teile derselben der Bewegungsrichtung parallel und entgegengesetzt gerichtet<sup>1)</sup>.

1) Vgl. Lamb, Hydrodynamics, Dritte Ausgabe S. 552.

1. L. Onsager, *Physikalische Zeitschrift*, 1926.
2. L. Onsager, *J Chem Phys*, 1931; *Science*, 1969.

“for the discovery of the reciprocal relations bearing his name, which are fundamental for the thermodynamics of irreversible processes”

【Characteristic length: Debye length, Bjerrum length, ion radius; relaxation time】 【Familiar one: independent motion】

12 December 1969, Volume 166, Number 3911

**SCIENCE**

## The Motion of Ions: Principles and Concepts

**“Anomalies” of strong electrolytes from the ideal additive behavior:**  
**Electrical conductivity, Freezing point depression, Electromotive force**

**This article is the lecture he delivered in Sweden, when he received the Nobel Prize in chemistry.**

pletely dissociated and the properties of a solution would be additive, not just over molecules, but even over the constituent ions. At higher concentrations, admittedly, one would have to allow for combination to form molecules or compound ions according to the mass-action law, as suggested by Ostwald (2). Nerst developed appropriate simple theories for the diffusion of electrolytes and for the variation of an electrode potential with the concentration of the ion discharged.

Such was the simple picture presented to me as a freshman chemist in 1920. In spite of some idealization it sufficed for a great many purposes; it eased many tasks no end and we were eternally grateful for that. However, every soon the journals rather than the lecture taught me about numerous observations which did not quite fit into the picture and of tentative explanations for the discrepancies. Whether the experimenters studied the electrical conductivities or the equilibrium properties like freezing point depressions and electromotive forces, significant deviations from the ideal additive behavior persisted to much lower concentrations than had been predicted according to the mass-action law from the measurements performed on more concentrated solutions at temperatures became known as the “anomalies” of strong electrolytes. In many ways the anomalies displayed conspicuous regularities; if one compared salts of the same valence type like NaCl and KNO<sub>3</sub>, the differences were typically small even at concentrations as high as 0.1 mole/liter. Suspicion centered on the long-range electrostatic forces between the ions.

Debye and Hückel finally succeeded in explaining the “anomalies” of the electrolytes. Their theory, which applies to the dilute region, is based on the assumption that the ions are completely dissociated and that the electrostatic forces between them are the only forces of importance. It is published here with the permission of the Nobel Foundation and will also be included in the complete volume of *Lars Onsager's Nobel Lecture* as well as in the series *Nobel Lectures in Chemistry* published by the Elsevier Publishing Company, Amsterdam and New York.

Copyright © 1969 by the Nobel Foundation.  
The author is J. Willard Gibbs Professor of Theoretical Chemistry at Yale University. This lecture was delivered in Stockholm, Sweden, January 1969, when he received the Nobel Prize in chemistry. It is published here with the permission of the Nobel Foundation and will also be included in the complete volume of *Lars Onsager's Nobel Lecture* as well as in the series *Nobel Lectures in Chemistry* published by the Elsevier Publishing Company, Amsterdam and New York.

# Landau's theory on electron evolution in solid

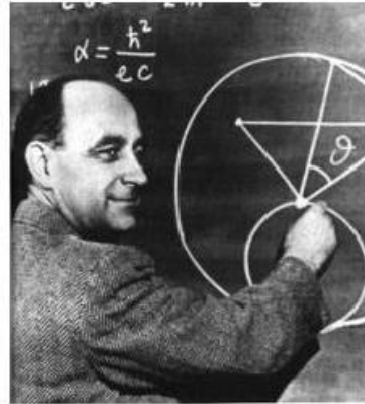
## Landau Fermi-liquid

## From Fermi gas to Fermi liquid

### WHY

particle kinetic  
description of  
electron transport  
still stands?

- ❑ **Excitation**
- ❑ **Propagation**
- ❑ **Scattering**

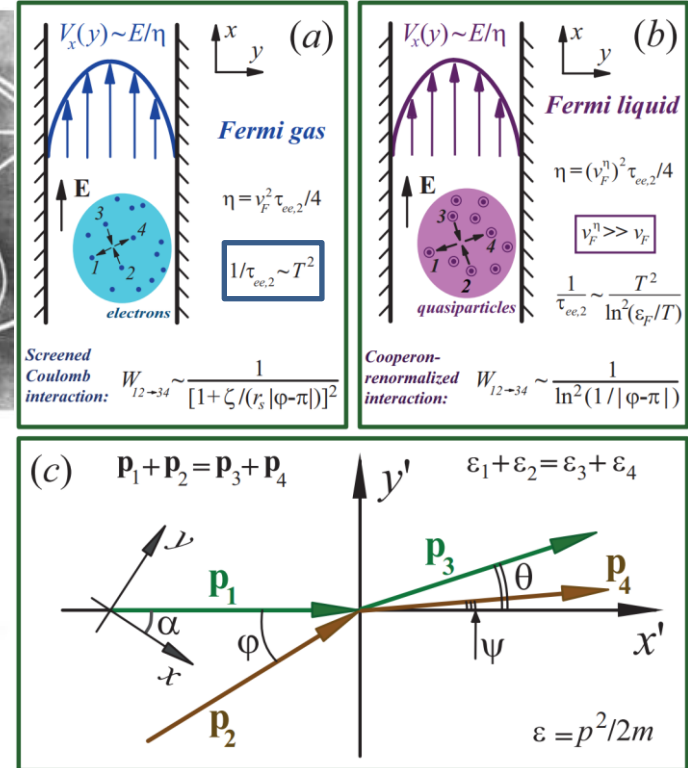


In 1956, Landau developed a theory of interacting spin-1/2 fermions<sup>1</sup>

The Landau-Fermi liquid theory successfully describes metals, nuclear matter, liquid He-3...

At low temperatures, the average excitation energy is  $\sim k_B T$

1. L. D. Landau, The theory of Fermi liquids, *Zh Eskp Teor Fiz*, 1956
2. P.S. Alekseev & A.P. Dmitriev, Viscosity of two-dimensional electrons, *Phys Rev B*, 2020



“for his pioneering theories for condensed matter, especially liquid helium”



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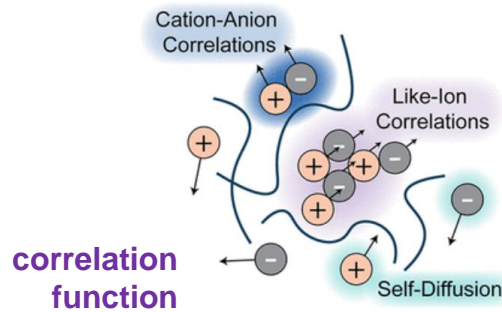
# Challenge II: From familiar to unfamiliar ones



## Electrolytes around interfaces

From **bulk, solid** to **liquid** interface

- | ➤ Nature           | ➤ Correlation          |
|--------------------|------------------------|
| ✓ <b>dissolved</b> | ✓ <b>steric</b>        |
| • hydrodynamic     | ✓ <b>electrostatic</b> |
| • chemical         | ✓ flow-induced         |



**Familiar ones – I** : nature and transport behavior of electrolytes in solution and electrons in solid



**Familiar ones – II** : transport of electrolyte transport at solid interface & phonon transport in solid

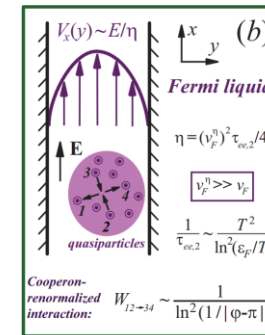
**WHETHER** kinetic description of **ion** transport at solid interface still stands?

- ❑ Nature of interface/phases
- ❑ Adsorption kinetics
- ❑ Ion transport dynamics

## Quantums in solids

From **phonon** to **electron**

- | ➤ Nature               | ➤ Correlation                                 |
|------------------------|---|
| ✓ <b>boson/fermion</b> | ✓ coherent                                    |
| ✓ lattice regulation   | ✓ <b>electro-</b> ( $e, \hbar$ ) <sup>2</sup> |
| ✓ wave nature          | ✓ <b>magneto-</b> ( $\uparrow, \downarrow$ )  |



**quasi-particle picture**

**WHETHER** kinetic description of **phonon** transport still stands?

- ❑ Total no. conservation
- ❑ Frequency domain width
- ❑ Inter-quasi-particle scattering



# Problem A: Electrolyte transport at interface (S)

⊕ ⊖ Cation | Anion

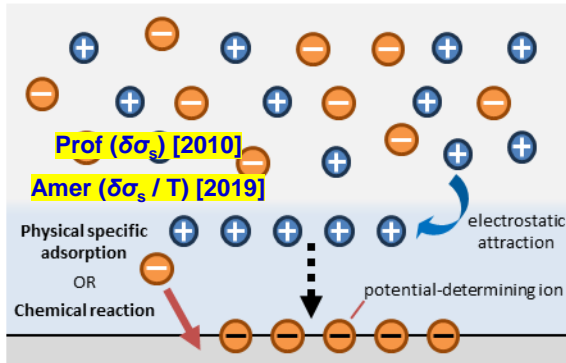
■ □ Solid | liquid region

□ Diffuse layer region

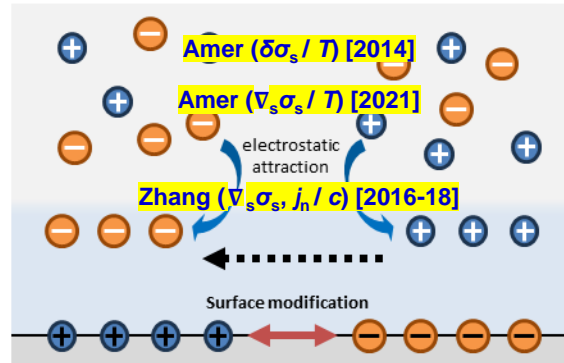
⋯→ Polarization field

→ Dominant mechanism

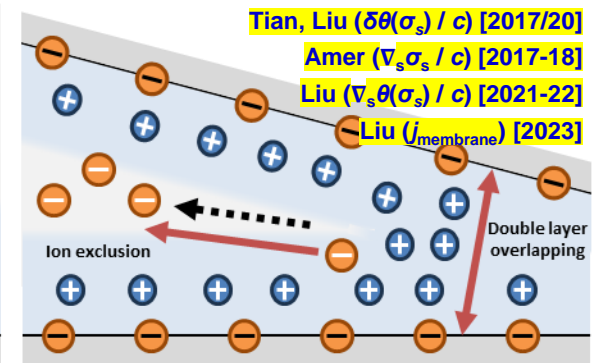
↪ Secondary mechanism



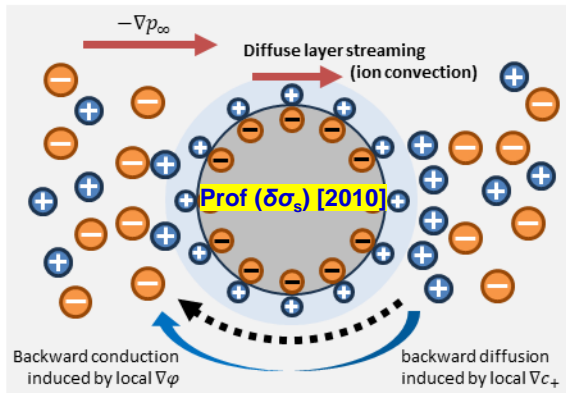
Formation of electric double layer



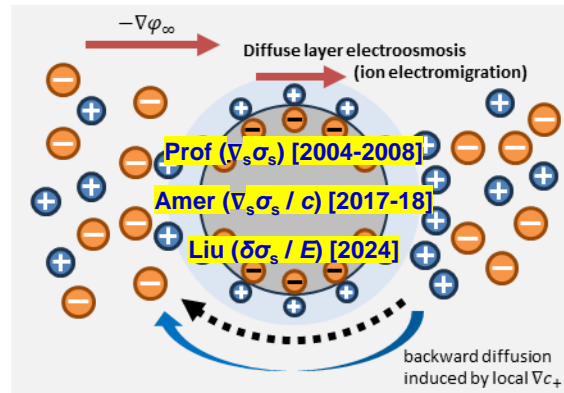
Chemical inhomogeneity



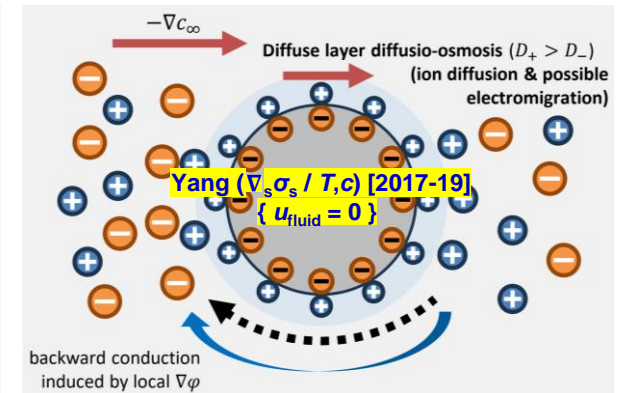
Confinement inhomogeneity



Pressure gradient upon an enclosure



Electric field upon an enclosure



Concentration gradient upon an enclosure



# Challenge: Electrolyte transport at interface (L)

⊕ ⊖ Cation | Anion

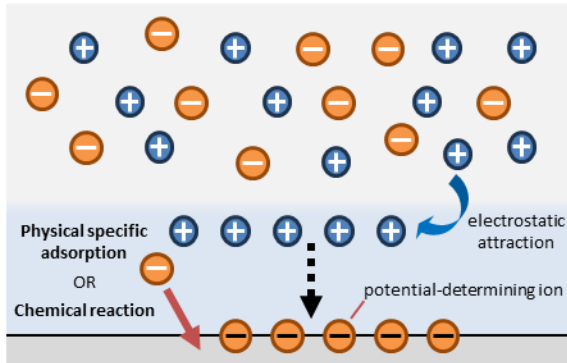
■ ■ ■ Solid | liquid | liquid regions

■ ■ Diffuse layer regions

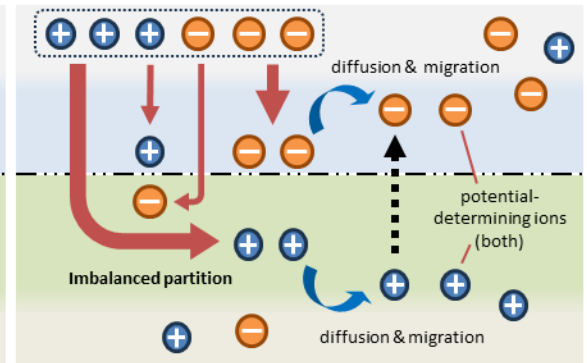
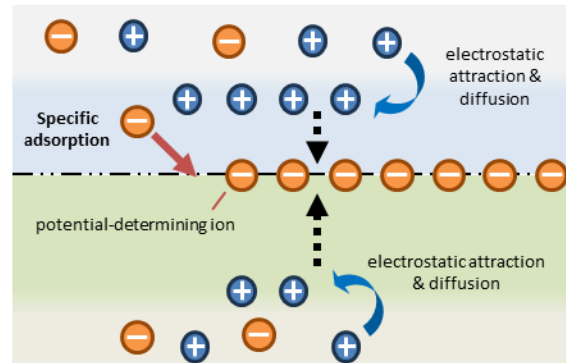
⋯→ Polarization field

→ Dominant mechanism

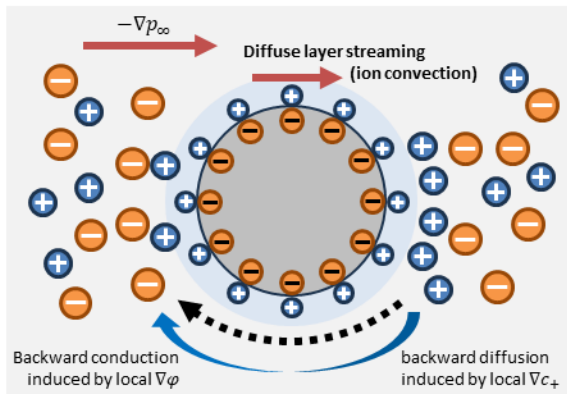
↪ Secondary mechanism



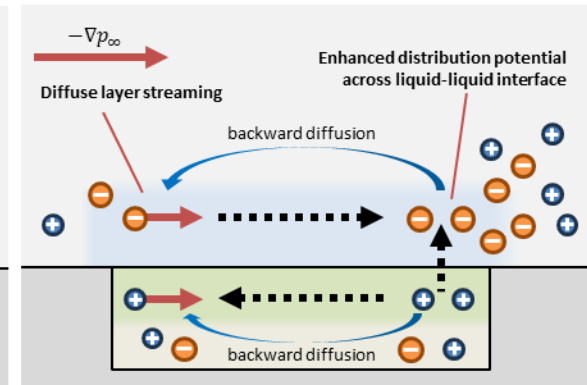
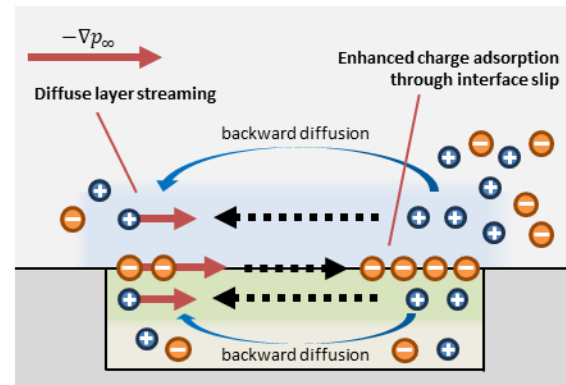
Formation of electric double layer



Feature 1: Complex charging mechanisms [Huang \( \$\delta\sigma\_s\$ \) \[2023-24\]](#)



Pressure gradient upon an enclosure

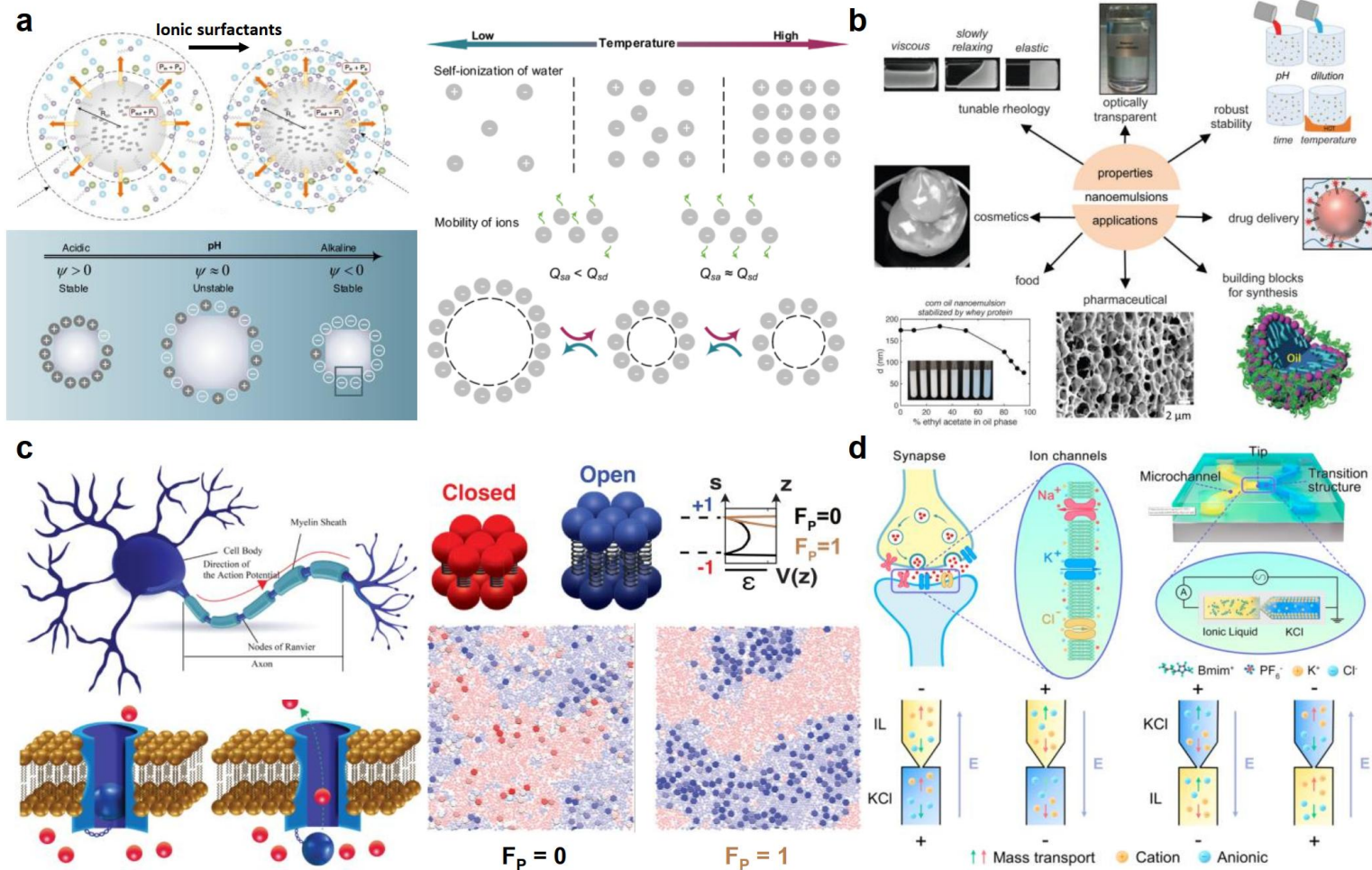


Feature 2: Slip-induced inhomogeneous charging

[Huang \(Ongoing\)](#)

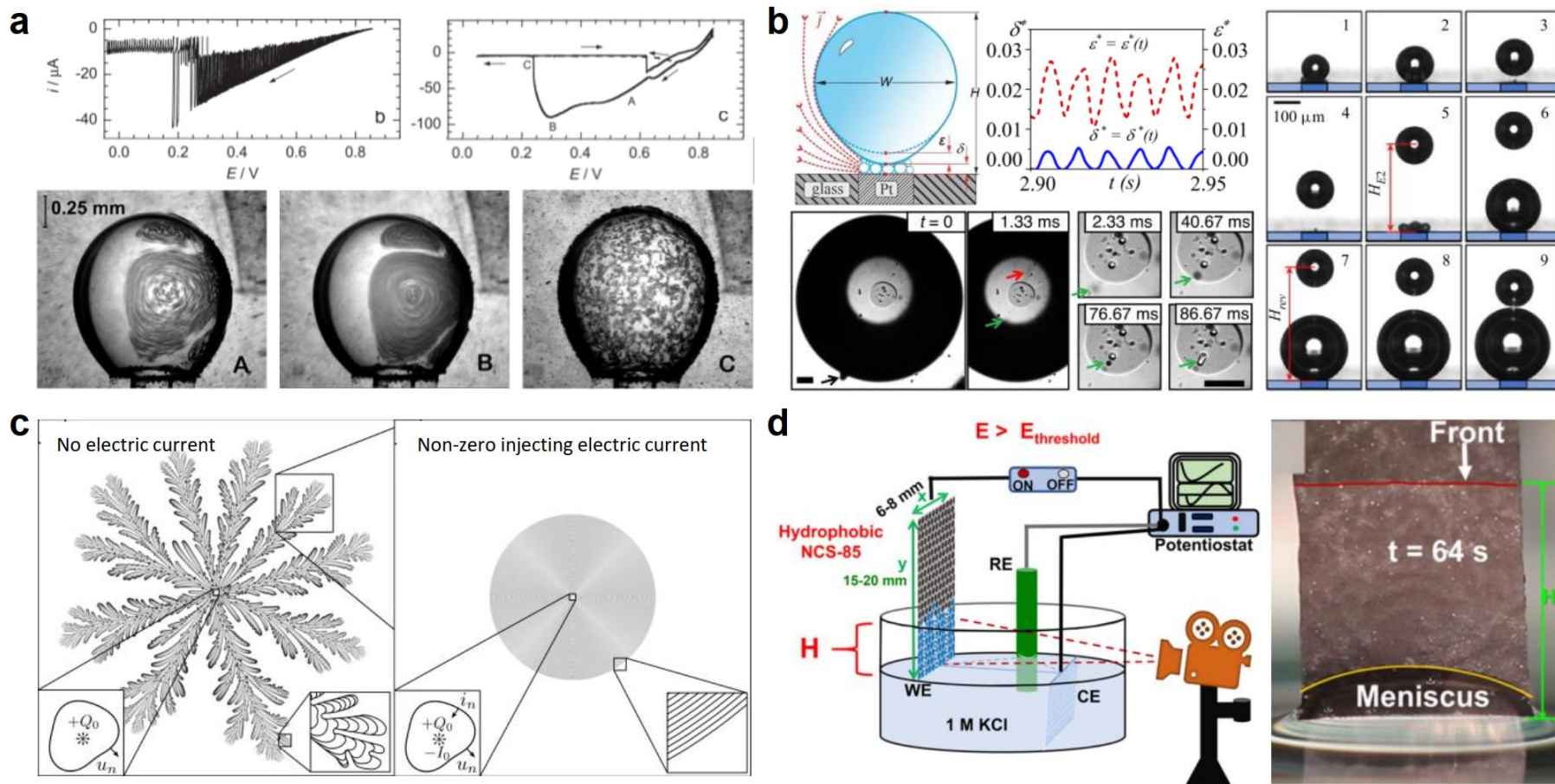
电动多相流体力学：带电液液界面 - 双侧双电层 - 额外对称性破缺 - 双侧极化耦合  
(界面吸附常数 + 介电常数比)

# Electrokinetic multiphase flow: interface charging



- a) Li *et al.*, *JCIS*, 2021; Ma *et al.*, *JCIS*, 2022 / 2024.  
 b) Gupta *et al.*, *Soft Matter*, 2016.  
 c) Torbati *et al.*, *Rev Mod Phys*, 2022; Suma *et al.*, *PRX Life*, 2024.  
 d) Li *et al.*, *Nano Lett*, 2024.

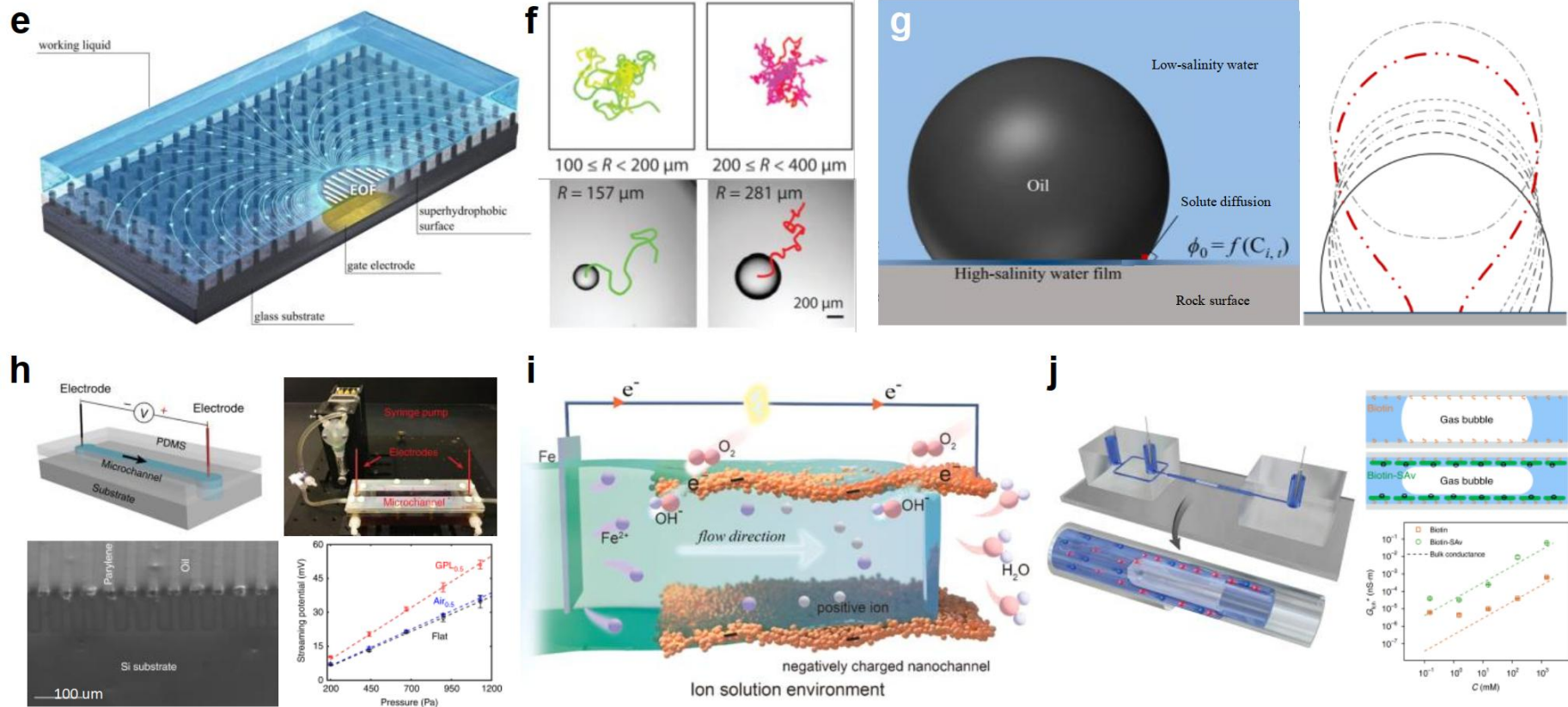
# Electrokinetic multiphase flow: field-driven flow



- a) Trojanek *et al.*, *Electrochem Comm*, 2017.
- b) Bashkatov *et al.*, *Phys Rev Lett / Phys Chem Chem Phys / Journal of Fluid Mech*, 2019/2022/2023/2024.
- c) Mirzadeh *et al.*, *Phys Rev Lett*, 2017.
- d) Pan *et al.*, 2023.



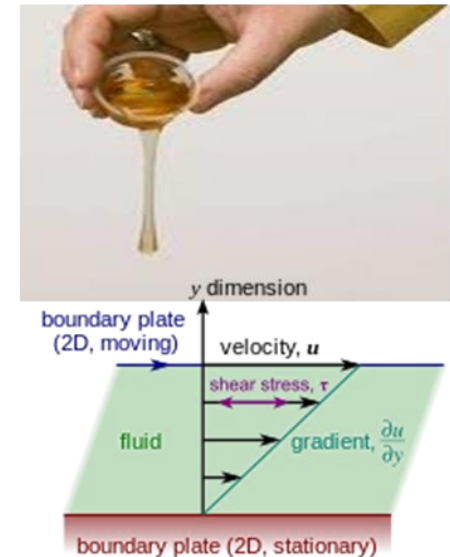
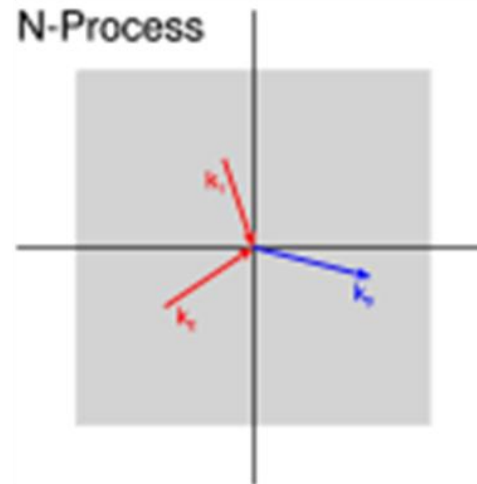
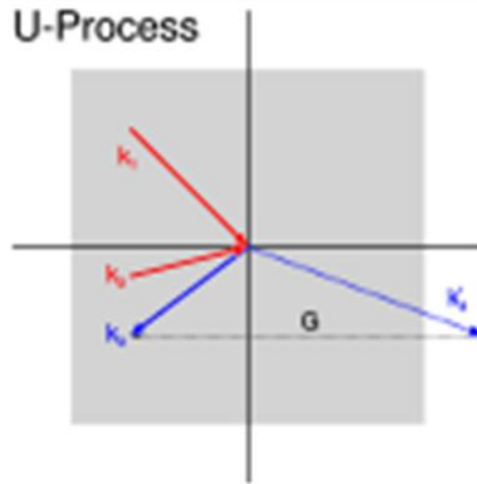
# Electrokinetic multiphase flow: ion-mediated transport



- e) Dehe *et al*, *Phys Rev Fluids*, 2020.
- f) Suda *et al*, *Phys Rev Lett*, 2021; Michelin, *Annal Rev Fluid Mech*, 2023.
- g) An *et al*, *Fuel*, 2022.
- h) Fan *et al*, *Nat Comm*, 2018.
- i) Li *et al*, *Nano Energy*, 2021.
- j) Ma *et al*, *Nat Comm*, 2020.

# Problem B: Transport in quantum systems (ph)

Room temperature  $\longrightarrow$  Low temperature



**Classical model**

Phonon  $\longleftrightarrow$  Phonon (U)

**Hydrodynamic model**

Phonon  $\longleftrightarrow$  Phonon (N)

**Viscous fluid model**

Molecular  $\longleftrightarrow$  Molecular

**Umklapp-induced resistance**  
(Fourier law)

**Boundary-induced resistance**  
(Hydrodynamic flow)

Prof (GEMC)  
[99-24]

Guo (Meso & Macro) [2013-18]; Miao, Ran (Meso) [2016-22]; Liu (Macro/Micro) [2023-24]

**Phonon hydrodynamics!**

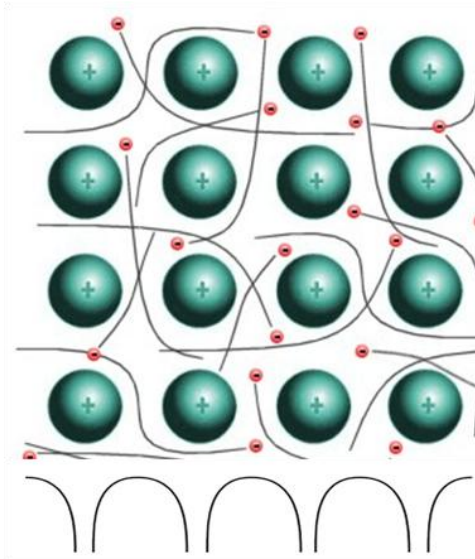
声子水动力学：守恒散射截面 - 动量守恒平衡态 - 声子集体运动 - 声子粘性  
(弛豫散射率 + 守恒散射率)



# Challenge: Transport in quantum systems (e)

Room/ultralow temperature  $\rightarrow$  Low temperature & low dimension & ultrapure

- Metal, Si|Ga[Al]As, Doped-S|BLG, Pt|PdCoO<sub>2</sub>
- SLG, semi-metal (MoP, WP<sub>2</sub>), topo-insulator



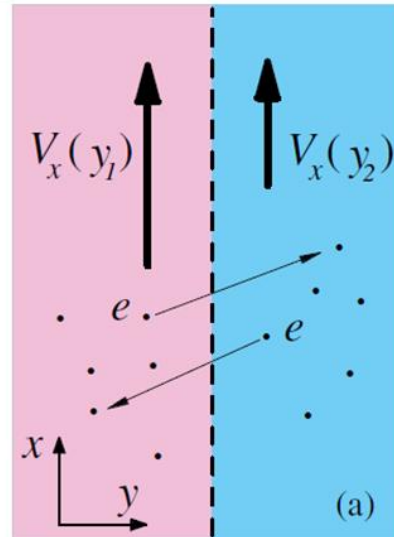
**Nearly Free electron model**

Electron  $\leftrightarrow$  Lattice\*

**Lattice-induced resistance**

(Ohmic flow)

Miao (Meso) [2017-22]



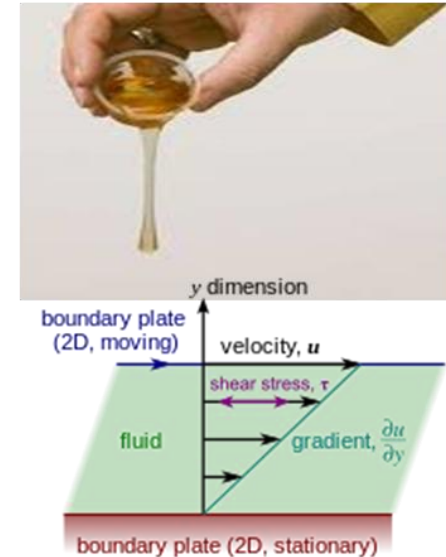
**Viscous electron model**

Electron  $\leftrightarrow$  Electron\*\*

**Boundary-induced resistance**

(Hydrodynamic flow)

Huang (Meso & Macro) [2017-21]; Meng (Macro) [2019]



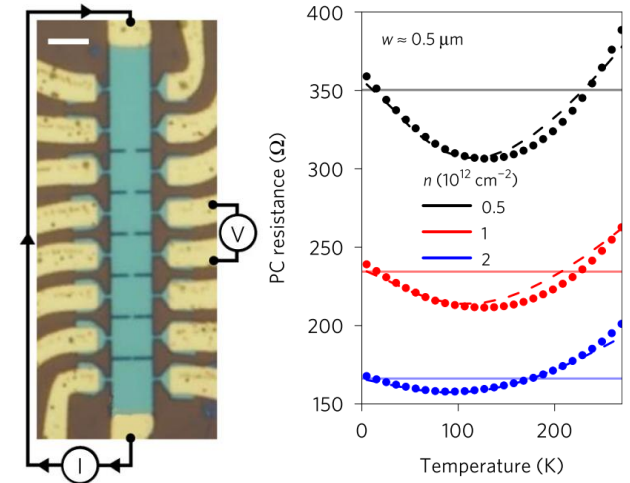
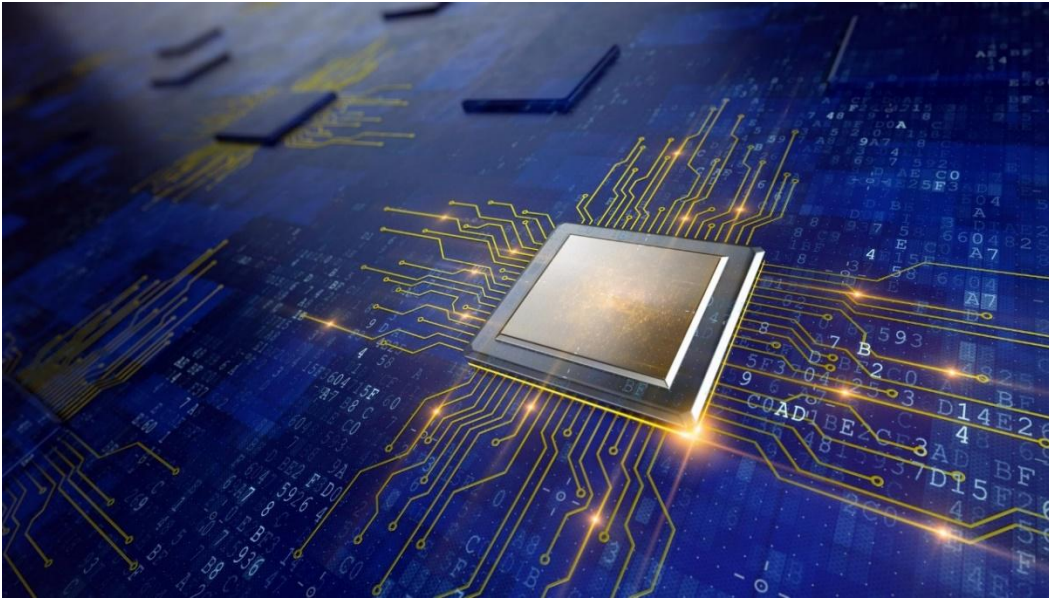
**Viscous fluid model**

Molecular  $\leftrightarrow$  Molecular

**Electron hydrodynamics!**

电子水动力学：守恒散射截面 - 动量守恒平衡态 - 电子集体运动 - 电子粘性  
(弛豫散射率 + 守恒散射率)

# Electron hydrodynamics: transit – $\sigma_e$



**Super-ballistic flow** in confined graphene

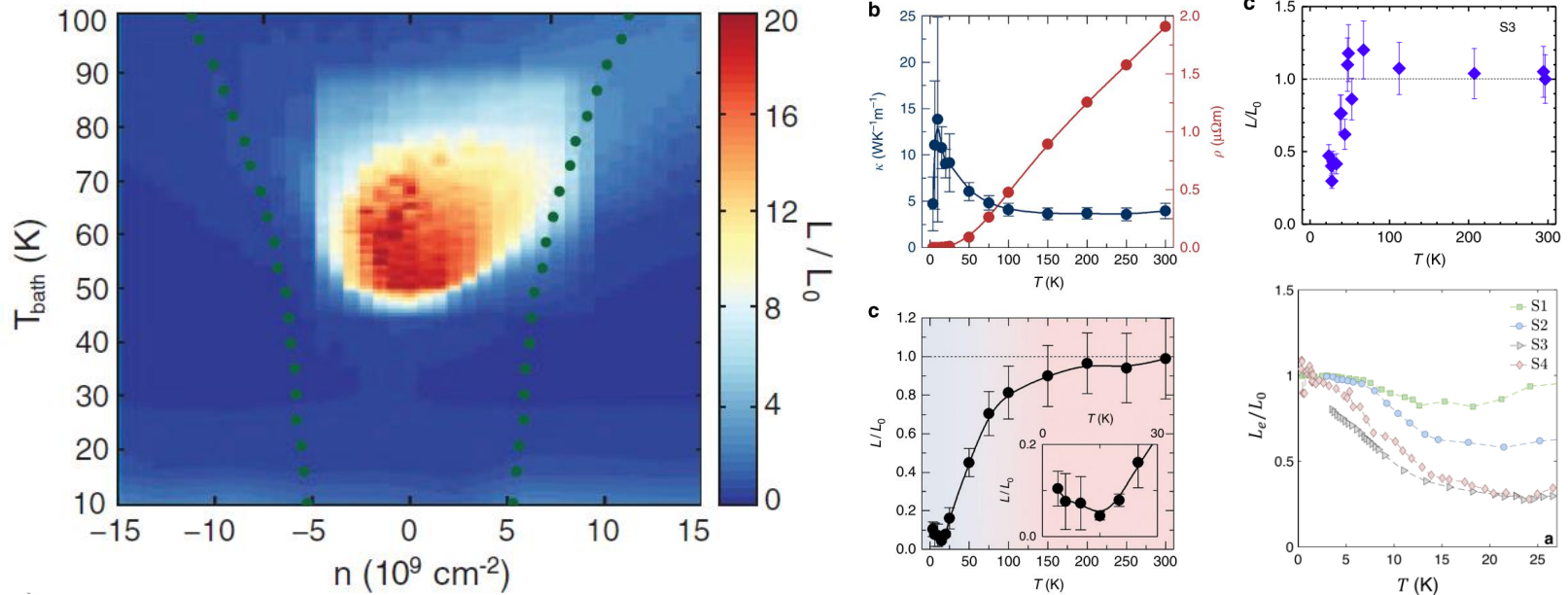
**High** electron conductivity

**Low** heat generation →  
**High** current transport capability

**High** cutoff frequency →  
**Low** switching time of the transistor

- ✓ System: 2D materials, ultra-pure, microscale and low-T
- ✓ Phenomena: high mobility, may improve performance of semiconductors

# Electron hydrodynamics: energy – $ZT_e$



**Breakdown of W-F law** in graphene (left) &  $WP_2$  (mid) & MoP (r-top) & Sb (r-bot)

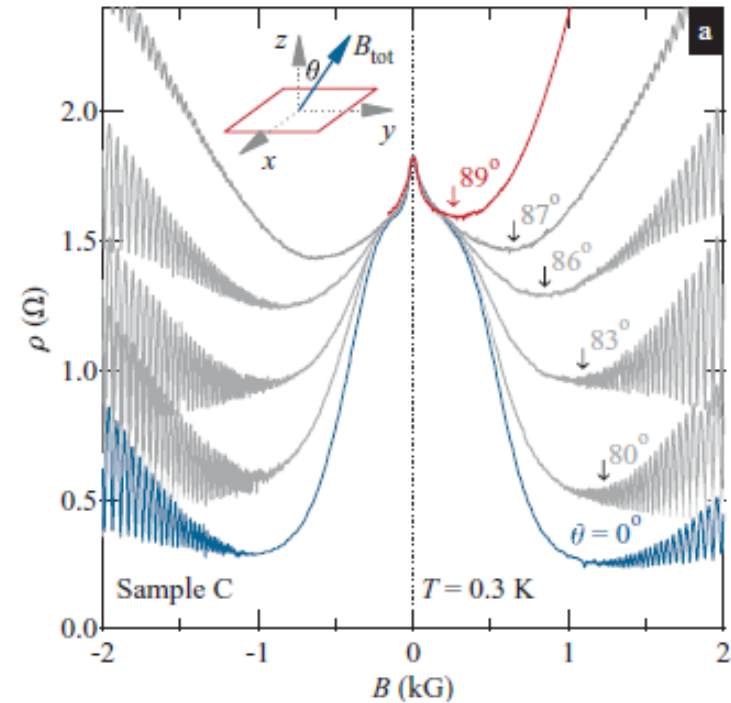
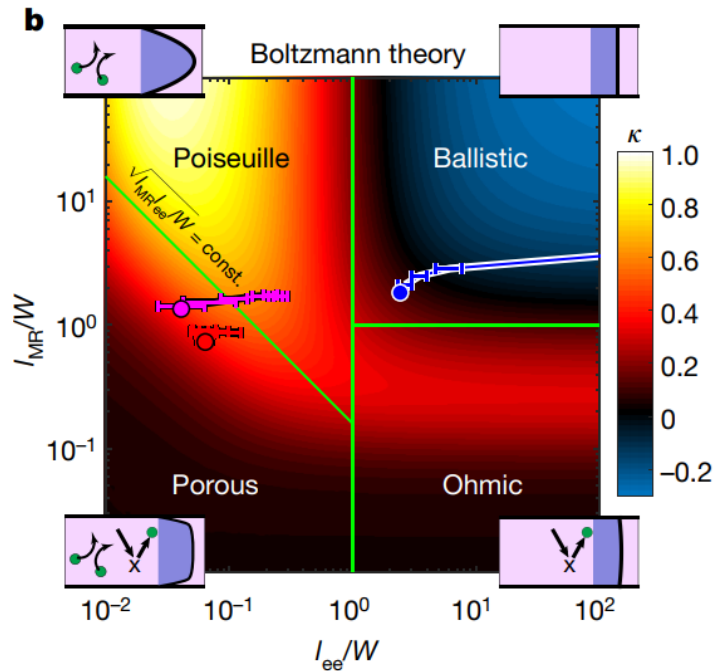
$$ZT = S^2 / \mathcal{L} \quad \mathcal{L} \equiv \frac{\kappa_e}{\sigma T} = \frac{\pi^2}{3} \left( \frac{k_B}{e} \right)^2 \equiv \mathcal{L}_0$$

higher/lower  $\kappa_e$  than W-F law

- ✓ System: 2D material, ultrapure, low T
- ✓ Phenomena: Anomalous transport of 2D electrons
- ✓ Impact: low heat loss in thermoelectric materials

1. Crossno J. *et al.* *Science*, **351**: 1058-1061, 2016.
2. Gooth, J., *et al.*, *Nat Comm.* **9**(1): 4093, 2018.
3. Jaoui, A., *et al.*, *npj Quantum Materials*, **3**(1): 64, 2018.
4. Kumar, N., *et al.*, *Nat Comm*, **10**(1): 2475, 2019.
5. Jaoui, A., B. Fauqué, and K. Behnia, *Nat Comm*, **12**(1): 195, 2021.

# Electron hydrodynamics: info – $\sigma_e(B,T)$



Conduction regime with different mechanisms  
(confinement)

Giant negative magneto-resistance in GaAs/AlGaAs  
(magnetic field)

**Magnetic field will dramatically impact the electric resistance in confined materials.**

1. Hatke A.T. *et al. Phys Rev B*, **85**: 081304, 2012.
2. Alekseev P. S. and Semina M. A. *Phys Rev Lett*, **98**: 165412, 2018.
3. Chandra M, Kataria G, Sahdev D, *et al. Phys Rev B*, **99**: 165409, 2019.
4. Mandal I, Lucas A. *Phys Rev B*, **101**(4): 045122, 2020.



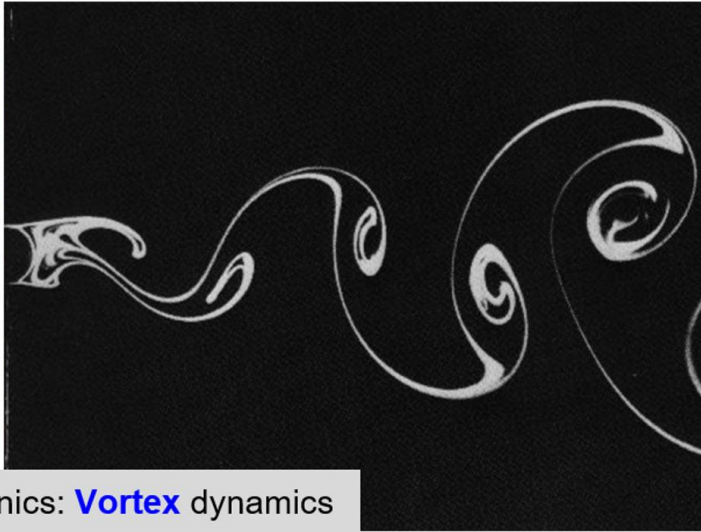
# Contents

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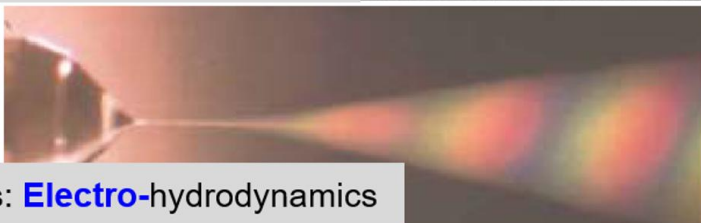
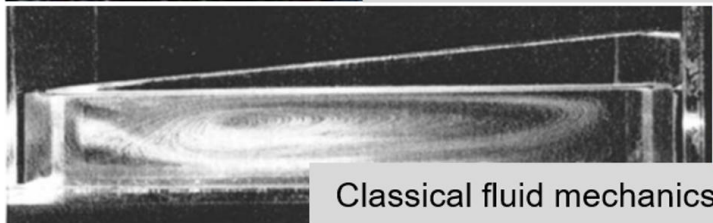
- From simple to complicated: find connections and differences
  - *“Where to start ... with something new”*
  - “Decouple” charge correlation: learn from TWO figures
  - Personal experience A: Electrolyte transport at interfaces
  - Personal experience B: Transport in quantum systems
- **Complex hydrodynamics as X-paradigm: What, Why, How, and Which?**
  - **Example: Emergence of quantum hydrodynamics**
  - **New physics: “More SCALES at INTERFACE is Different”**
  - **Hallmark: Merging kinetic behaviors into hydrodynamics**



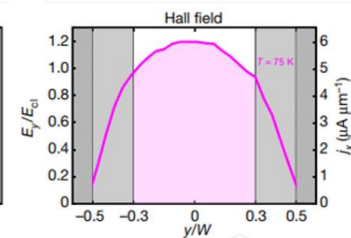
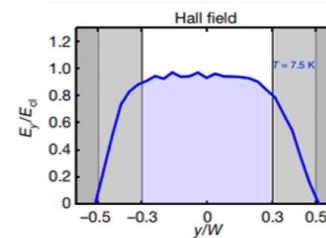
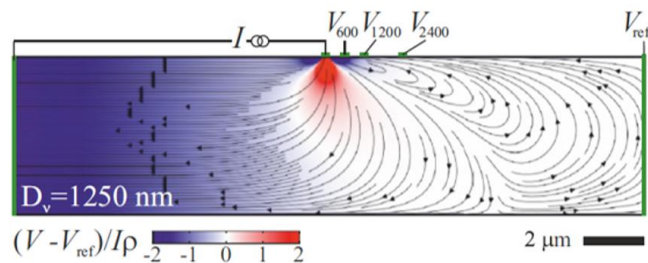
# From one paradigm (electro-hydrodynamics) ...



Classical fluid mechanics: **Vortex** dynamics



Classical fluid mechanics: **Electro**-hydrodynamics



Novel phenomena in 2D materials: **Electron backflow** | From **Ballistic** to **Viscous** regime



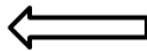
# And another paradigm (physical kinetics) ...

- Boltzmann transport equation (BTE) – semi-classical description

collective  
moving  
velocity

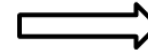
$$\boxed{\frac{df}{dt}} = \frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{r}} + \frac{\mathbf{F}}{m} \cdot \frac{\partial f}{\partial \mathbf{v}} = \boxed{C(f)} \left\{ \begin{array}{l} \text{Resistive scattering} \quad f_0(\mathbf{r}, \varepsilon_k) \\ \text{Conservative scattering} \quad f_0(\mathbf{r}, \varepsilon_k - \mathbf{u} \cdot \mathbf{p}) \end{array} \right.$$

Change of  
particle state



Particle  
scattering

Different  
mechanisms



Different (quasi-)  
equilibrium states

$f$ : the non-equilibrium distribution function of the particle cluster around  $(\mathbf{r}, \mathbf{v})$

**Solving BTE: Deterministic | Stochastic (particle nature)**

**Upscaling BTE: Hydrodynamic description (macroscopic)**

**Beyond BTE:** wave nature, strong correlation, scattering rate

Coherence: Quantum transport in low-D system

Localization: Strong disordered/correlated system

Super/Magneto/Topo: Strong (spin/Coulomb) correlated system



1. Rammer. *Quantum transport theory*. Perseus, 1998.
2. Datta. *Quantum transport – Atom to transistor*. Cambridge University Press, 2005
3. Nazarov. *Quantum transport – Introduction to nanoscience*. Cambridge University Press. 2009.
4. G. Chen. *Nanoscale energy transport and conversion*, Tsinghua University Press, 2014



# Into X-paradigm (quantum hydrodynamics) ...

- Boltzmann transport equation (BTE) – Chapman-Enskog expansion

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{r}} + \frac{\mathbf{F}}{m} \cdot \frac{\partial f}{\partial \mathbf{v}} = C(f) \quad f = f^{(0)} + \epsilon f^{(1)} + \epsilon^2 f^{(2)} + \dots$$

- Classical N-S equation

$$\frac{D\mathbf{P}}{Dt} + \nabla \cdot \boldsymbol{\sigma} = \rho \mathbf{g}$$

$$\boldsymbol{\sigma} = p\mathbf{I} - \mu \left( \frac{2}{3} (\nabla \cdot \mathbf{u}) \mathbf{I} + \nabla \mathbf{u} + (\nabla \mathbf{u})^T \right)$$

- Phonon N-S equation

$$\frac{\partial \mathbf{q}}{\partial t} + \nabla \cdot \mathbf{Q} = -\frac{\mathbf{q}}{\tau_R}$$

$$\mathbf{Q} = \frac{1}{3} v_g^2 e \mathbf{I} - \frac{1}{5} v_g^2 \tau_N ((\nabla \cdot \mathbf{q}) \mathbf{I} + \nabla \mathbf{q} + (\nabla \mathbf{q})^T)$$

- Electron N-S equation

$$\frac{\partial \mathbf{P}}{\partial t} + \nabla \cdot \mathbf{T} = -\frac{\mathbf{P}}{\tau_{MR}} + m^* \mathbf{F}_{\text{macro}} \quad \mathbf{T} = \mathcal{P} \mathbf{I} - \mu_e ((\nabla \cdot \mathbf{u}) \mathbf{I} + \nabla \mathbf{u} + (\nabla \mathbf{u})^T)$$

1. Y.Y. Guo. 非傅里叶导热的宏观声子输运模型及非平衡热力学. 博士学位论文, 2018.
2. Y.F. Huang. 微纳尺度低维电子输运的水动力学研究. 本科毕业论文, 2019.

# “More SCALES at INTERFACE is Different”



## Electrolytes around interfaces

From solid to **liquid**

$$\begin{array}{l} \lambda_D, \zeta_s/\sigma_s, [\alpha_h] \\ T_s^e, [\beta_E, Pe] \\ j_s, [j_n, \nabla_s \sigma_s] \end{array} \longrightarrow \begin{array}{l} \lambda_{D,i}, \sigma_s, \Phi_\infty(r_i, \epsilon_i), [\alpha_{h,i}] \\ T_{s,i}^e, [\beta_E, Pe_i, Ca_\theta] \\ j_{s,i}, [j_n, \nabla_s \sigma_s, \nabla_s \Phi_\infty] \end{array}$$

- **Mean distance (background)**
  - $n_\infty d_M^3 \sim 1$
- **Radius of ion (“coherence”)**
  - radius variation **hydration**
- **Debye length (screening)**
  - $\epsilon (\delta\phi/\lambda_D)^2 \sim (\delta n_\infty) k_B T$
  - concentration **ratio**
  - solvent permittivity **ratio**
- **Bjerrum length (correlated)**
  - $k_B T \sim e^2/\epsilon d_B$
  - solvent permittivity **ratio**
- **Relaxation time (“scattering”)**
  - $\tau_{diff} \sim \lambda_D^2 / (k_B T \mu_m)$
  - frictional force  $[\mu, E, \dots]$
  - viscosity **ratio** (gas/self?)
  - dissociation **constant** (weak?)

## Quantums in solids

From phonon to **electron**

$$[l_{tri}], l_N, l_R, W, [l_{dis}] \longrightarrow [\lambda_D], l_N, l_R, W, [l_{dis}] \\ [E_F, U_{corr}, \beta_{E,B}]$$

- **Lattice periodicity (background)**
  - interaction energy **uniformity**
  - steric effect (entropy)
- **Energy wavepacket (coherence)**
  - wave vector [volume] **Fermi surface**
  - frequency [life] **Fermi energy**
- **Debye length (screening)**
  - $\epsilon (\delta\phi/\lambda_D)^2 \sim (\delta n_\infty) k_B T$
  - concentration **prescribed**
  - lattice permittivity
- **Bjerrum length (correlated)**
  - $k_B T \sim e^2/\epsilon d_B$
  - lattice permittivity **correlation**
- **Relaxation time (scattering)**
  - $1/\tau_i \sim \sum_{p,\Omega} |\Delta v| |d\sigma/d\Omega| (f_1 f_2 - f_1' f_2')$
  - scattering events: X-X (**N/U**), **X-Y**

General Nature  
(physics)

Transport/Evolution  
(kinetics)

# “Interface” from the perspective of multiscale ...

LGCA: lattice gas cellular automaton

LB: lattice Boltzmann model

CG: color gradient model

PP: pseudo-potential model

MD: molecular dynamics

DPD: dissipative particle dynamics

MP: material point method

PIC: particle-in-cell method

MC: Monte-Carlo method

SPH: smoothed particle hydrodynamics

FT: front-tracking method

ALE: arbitrary Lagrangian-Eulerian method

IB: immersed boundary method

VoF: volume-of-fluid model

LS: level-set model

FE/PF: free energy / phase field model

NSK: Navier-Stokes-Korteweg equation

BTE: Boltzmann transport equation

PBE: population balance equation

MFM: multi-fluid model

MTE: momentum transport equation

## Lattice-based

- LGCA, LB
- CG, PP

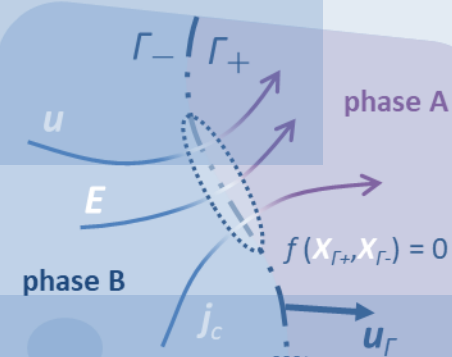


## Meshless

- MD, DPD
- MP\*, PIC\*
- MC, SPH

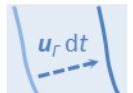


## Evolution of phase interface



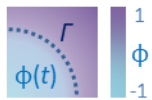
## Interface tracking

- FT, ALE
- IB



## Interface capturing

- VoF, LS
- NSK
- FE/PF



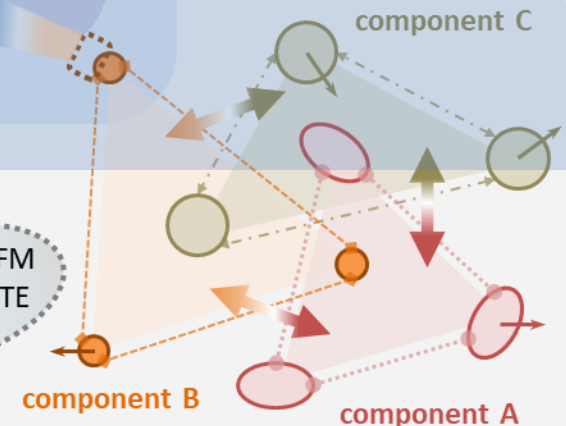
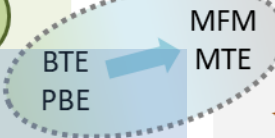
## particle A



## particle B



## Motion of representative particles



## Collective behavior of mixture

新物理源自新研究对象(视角): 新界面 → 新尺度 → 新机理 → 新描述 → 新方法





# X-paradigm: What, Why, How, and Which ?

- 何为**力学 (工科基础)** 相关的交叉学科？
  - 《科学革命的范式》范式及其**转换**
  - 《论技术科学》例外**来自/创造**工程需求
  - 交叉特征：**界面**接触 + **内核**融合 + **进化**独立
  - 现实投影：工业**需求** + 大牛**烙饼** + 众人**推广**
  - 典型案例：软物质物理，计算神经科学
- 原学科范式 – 力学 / 物理 / 化学
  - 经典[电]流体力学：[electro-]hydrodynamics
  - ✓ 电化学/胶体科学 (离子吸附)：chemical kinetics
  - ✓ 与/或，凝聚态物理 (粒子散射)：physical kinetics
- 新学科范式 – 或仅作为名词归纳
  - 复杂流体力学：complex (soft) hydrodynamics
  - 研究对象：含**内部额外 (强关联)** **自由度**的流体体系
  - 复杂机理举例：奇界面，多物理，子结构 **“序参数”**
    - 子分支 1：multiphase electrokinetic hydrodynamics
    - 子分支 2：quantum hydrodynamics
    - 子分支 3：soft flowing matter physics (micro-rheology)
- 关键图像 – merge kinetics into hydrodynamics
  - 传统手段：**物理实验** → **理论模型** → 数值模拟(实验)
  - 新兴手段：**理论建构** 与 **数据处理** 的结合

## 电动多相流体力学    量子水动力学

- a) 出现新“界面”：带电液液界面 | 守恒散射截面
- b) 涌现新“尺度”：介电常数比 | 守恒散射率
- c) 新“研究对象”：**电动双侧耦合** | **量子集体运动**
- d) 引入新“机理”：**界面极化** | **量子粘性**
- e) 要求新“描述”：有效边界 | 水动力学
- f) 亟需新“方法”：摄动展开 | 升尺度展开

### 多尺度系统的对策与挑战

- ↪ **奇界面**：摄动理论-时空分区，**参数要求高、难延拓**
  - ↪ **子结构**：粗粒理论-代表单元，**大尺度模型、难整合**
  - ↪ **多物理**：有效理论-机理提取，**时空强关联、难解耦**
  - ↪ **反问题**：低维理论-去除冗余，**优化成本高、难规划**
- 涌现/演生**  
**(emergence)**

系统不同尺度的模型可能完全不同  
并且互相几乎“不可通约”  
多尺度模拟通常仍只是“黑箱”！



# Last but not at least ...

- From simple to complicated: find connections and differences
  - **“Where to start ... with something new”**
  - “Decouple” charge correlation: learn from TWO figures
  - *PhD proposal*: Electrolyte transport at two-liquid interfaces
  - *Undergrad thesis*: Transport in weakly-correlated electron systems
- **Complex hydrodynamics as X-paradigm**: What, Why, How, and Which?
  - **Example**: Birth of Huang’s Undergrad/PhD proposals
  - **New physics**: more “insights beyond **local consensus**” is different
  - **Hallmark**: The first paper (易上手 – 避免强耦合, 抓本质 – 敢于做假设)
  - **Positive feedback requiring**: 良师益友, 敢想敢干, 博观约取, 厚积薄发



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**Thank you**